Implications of BNL measurement of *δaµ* **on a class of scalar leptoquarkinteractions**

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Abstract. Recently BNL have measured the muon magnetic moment anomaly with increased precision. The world average experimental value at present shows a discrepancy of $43(16) \times 10^{-10}$ from the standard model (SM) value. In this paper we investigate the implications of this difference on a class of scalar leptoquark interactions to SM quark–lepton pair. We find that for a leptoquark mass of a few hundred GeV the BNL muon anomaly can arise from couplings much smaller than even the electromagnetic coupling. We also find that the BNL value for the muon anomaly leads to an unambiguous prediction for the electric dipole moment of the muon and a bound on the flavor changing leptoquark coupling relevant for the rare decay $\mu \to e\gamma$.

The BNL collaboration has reported a new improved measurement of the muon magnetic moment anomaly [1]. The present average value shows a discrepancy of 43 (16) \times 10^{-10} from the estimated standard model (SM) value. A variety of new physics scenarios like extra gauge bosons, compositeness, extra fermions and supersymmetry have been considered to explain this discrepancy [2]. The muon anomaly had also been used in the past to constrain physics beyond the SM. For earlier works related to this subject see [3].

In this paper we shall analyze the possibility that the BNL muon anomaly could be due to elementary leptoquarks. Leptoquarks are scalar or vector particles in the 3 or 3^{*} representation of $SU(3)_c$ that couple to quark– lepton pairs. They can be in the triplet, doublet or singlet representation of $SU(2)_l$. They can also carry weak hypercharge. Leptoquarks occur in many scenarios beyond the SM, e.g. technicolor models, substructure models of quarks and leptons and string inspired grand unified models like E(6). Consider a scalar leptoquark which has the following Yukawa interactions $[4]$ to SM fermions:

$$
L^{np} = (g_{\text{L}ij}\bar{q}_{\text{R}}^{ci}\text{i}\tau_2 l_{\text{L}}^j + g_{\text{R}ij}\bar{u}_{\text{L}}^{ci}e_{\text{R}}^j)S_1 + \text{h.c.}
$$
 (1)

Here $g_{\text{L}ij}$ and $g_{\text{R}ij}$ are the leptoquark couplings to LH and RH leptons. i and j refer to quark and lepton generations. $\psi^c = C\bar{\psi}^T$ is the charge conjugated spinor. S_1 is second generation leptoquark in the 3[∗] representation of color. It is a weak isoscalar and has charge 1/3. The above interaction Lagrangian will generate a one loop correction to the magnetic moment [5] of muon which can be estimated from the following effective Lagrangian:

$$
L_{\text{eff}} = \sum_{i} i e m_{ui} I^{i}(0,0) \text{Re}(g_{\text{L}i2}^{*} g_{\text{R}i2}) \bar{\mu} \sigma_{\mu\nu} \mu F^{\mu\nu}, \quad (2)
$$

where $\text{Re}(g_{\text{L}i2}^*g_{\text{R}i2})$ means the real part of $(g_{\text{L}i2}^*g_{\text{R}i2}), m_{ui}$ is the mass of the $Q = 2/3$ quark of the *i*th generation and

$$
I^{i}(0,0) = \int \frac{\mathrm{d}^{4}l}{(2\pi)^{4}} \frac{1}{(l^{2} - m_{ui}^{2})[l^{2} - m_{ui}^{2}][l^{2} - m_{s}^{2}]}.
$$
 (3)

 m_s is the mass of the leptoquark $S₁$. From the above effective Lagrangian it follows that the one loop correction to the magnetic moment of the muon is given by

$$
\delta\mu = \sum_{i=1}^{3} 2iem_{ui}I^{i}(0,0)Re(g_{Li2}^{*}g_{Ri2}).
$$
 (4)

On evaluating the loop integral we find that

$$
I^{i}(0,0) = -\frac{i}{16\pi^{2}m_{s}^{2}} \left[-\frac{1}{1-r_{i}} - \frac{\ln r_{i}}{(1-r_{i})^{2}} \right],
$$

where $r_i = m_{ui}^2/m_s^2$.

The contribution of the up quark to $\delta \mu$ can be neglected firstly because its mass is small and secondly because it involves off-diagonal couplings between the first and second generation. The contribution of the top quark however can be comparable to that of the charm quark because its large mass can partly compensate for the small off-diagonal leptoquark couplings between the second and third generation fermions. We have

$$
a_\mu^{np} \approx \sum_{i=2}^3 {m_{ui} m_\mu \over 4 \pi^2 m_s^2} \left[-{1 \over 1-r_i} - {\ln r_i \over (1-r_i)^2} \right]
$$

$$
\times \text{Re}(g_{\text{L}i2}^* g_{\text{R}i2}). \tag{5}
$$

 S_1 being a second leptoquark, its couplings g_{L22} and g_{R22} to second generation fermions are expected to be strongest, whereas the flavor violating couplings q_{L32} and g_{R32} are expected to be hierarchically smaller. In the numerical estimates presented here we shall assume that $\text{Re}(g_{\text{L}32}^*g_{\text{R}32}) \approx .01 \text{Re}(g_{\text{L}22}^*g_{\text{R}22})$ i.e. a Cabibbo type suppression. We find that for these values of the flavor offdiagonal couplings the top contribution is much smaller than the charm contribution. Since the BNL value of the muon anomaly allows one to find bounds on the leptoquark mass for given values of the couplings but not both, we shall introduce an effective scale Λ for expressing the leptoquark contribution to a_{μ}^{np} where

$$
\frac{1}{\Lambda^2} = \frac{\text{Re}(g_{\text{L22}}^* g_{\text{R22}})}{m_s^2} \left[-\frac{1}{(1-r_2)} - \frac{\ln r_2}{(1-r_2)^2} \right].
$$
 (6)

The introduction of Λ can be looked upon as a convenient normalization of the leptoquark contribution to a_{μ}^{np} . Equating a_{μ}^{np} to the central value for δa_{μ} we get $\Lambda \approx 890 \,\text{GeV}$. The new value of δa_μ is important because of two reasons. First it represents a large discrepancy (2.6σ) effect) between the SM value and the measured value. Second the new measurement has an error one third of the combined previous data. Both these factors can be taken into account by determining 95% CL limits on Λ . For the present world average value of δa_{μ} these bounds are given by 678 GeV $\leq \Lambda \leq 1760$ GeV. The ultimate goal of the experiment [2] is to reduce the error to $\pm 4 \times 10^{-10}$, about a factor of 3.5 times better than the new BNL result. Even the inclusion of already existing data from the 2000 run is expected to reduce the statistical error by a factor of 2. If the central value and other errors are unaffected this would improve the bounds on Λ to 764 GeV $\leq \Lambda \leq 1123$ GeV. We would like to emphasize that Λ should not be interpreted as the leptoquark mass. The 95% CL bounds on the couplings for a given value of the leptoquark mass can be obtained from (6) by substituting the bounds for Λ given above. For example if $m_s = 300 \,\text{GeV}$ the 95% CL bounds on the couplings are $.003 \leq \text{Re}(g_{\text{L22}}^* g_{\text{R22}}) \leq .020$.

The leptoquark contribution to $(g-2)_{\mu}$ involves a chirality flip on the internal quark line. This causes the resulting expression to be proportional to $\text{Re}(g_{\text{L}}^{*}g_{\text{R}})$. The most stringent bound on $|g_{\text{L}}g_{\text{R}}|$ for first generation leptoquarks arises from the helicity suppressed decay mode π \rightarrow $e \nu_e$ and is given by $|g_{\text{L}}g_{\text{R}}| \leq m_{lq}/30 \,\text{TeV}$ [4,6]. The analogous decay $K \to \mu \nu_e$ for second generation leptoquarks however involves flavor changing couplings. Further, since helicity suppression is not that stringent for second generation fermions we expect the bound on $\text{Re}(g_{\text{L22}}^*g_{\text{R22}})$ to be much weaker than it is for the first generation. Conservatively even if we assume it to be of the same order as for first generation it would be consistent with the bounds $.003 \leq \text{Re}(g_{\text{L}_2/2}^* g_{\text{R}_2/2}) \leq .020$ derived in this paper from the muon anomaly.

The direct bounds on m_{lq} set by Tevatron and HERA [7] assumes that $g_{lq} \approx e$. The Tevatron bounds arise from pair production of leptoquarks. These bounds depend on the color and electroweak quantum numbers of the leptoquark. For a second generation scalar leptoquark having the quantum numbers of S_1 the Tevatron bound is $m_{lq} > 200 \,\text{GeV}$. This limit depends upon the assumption that $B(\mu, q) = 1$. For $B(\mu, q) = 0.5$ the bound becomes 180 GeV. On the other hand HERA sets bounds on leptoquark masses from single production. The bound on a second generation S_1 leptoquark is $m_{lq} > 73 \,\text{GeV}$. The bounds on the leptoquark mass derived in this paper from the muon magnetic moment anomaly are therefore stronger than the direct bounds set by HERA and Tevatron.

We shall now consider the implications of the BNL result on the electric dipole moment (EDM) of the muon. Since $g_{2\text{L}}^* g_{2\text{R}}$ can in general be complex the imaginary part of $g_{2\text{L}}^*g_{2\text{R}}$ can contribute to the EDM of the muon. In the estimates presented below we shall consider the flavor diagonal leptoquark couplings only and ignore the top contribution which is expected to be numerically smaller. We find that the EDM of the muon is given by

$$
d_{\mu}^{\gamma} \approx 2iem_{c}I(0,0)\mathrm{Im}(g_{\text{L22}}^{*}g_{\text{R22}}), \tag{7}
$$

where Im means the imaginary part of the relevant quantity. If the BNL discrepancy between the measured and the SM value of the muon magnetic moment anomaly is due to leptoquarks then one can derive an order of magnitude value for the EDM of the muon. In the SM the EDM of leptons vanishes to three loops and is predicted to be of the order of $(1.6m_l/MeV) \times 10^{-40}e$ cm [8]. The SM contribution to the EDM of leptons is therefore far too small to be observed in any experiments to be performed in the near future. So any observed value of the EDM of leptons must be due to new physics. In the SM the small value of CP violation follows from the small value of the CKM angles but not the phase. Assuming that the phase factor sin δ for leptoquark couplings is of order one we have $\text{Im}(g_{2\text{L}}^*g_{2\text{R}}) \approx \text{Re}(g_{2\text{L}}^*g_{2\text{R}})$. The value of $I(0,0)\text{Re}(g_{2\text{L}}^*g_{2\text{R}})$ can be estimated from the discrepancy in $(g-2)_{\mu}$ by BNL and is given by 6.8×10^{-9} . It then follows from (6) that the EDM of muon is of the order of 4.1×10^{-22} e cm. The muon anomaly reported by BNL therefore leads to an almost (except for a choice of phase) unambiguous prediction for the EDM of muon. The predicted value however is three orders of magnitude smaller than the present experimental upper bound of 3.7×10^{-19} e cm [7] on the EDM of muon.

The BNL measurement also has important implications for the lepton flavor violating decay $\mu \to e\gamma$. Leptoquarks can have tree level flavor violating couplings to quark–lepton pairs and they can induce the rare decay $\mu \to e\gamma$ at the one loop level. It can be shown that the relevant effective Lagrangian is given by

$$
L_{\text{eff}} = \xi \bar{e} \sigma_{\mu\nu} \mu F^{\mu\nu} + \text{h.c.},\tag{8}
$$

where $\xi = \mathrm{i} e m_c I(0,0) \mathrm{Re}(g_{\text{L21}}^* g_{\text{R22}})$. It then follows that the decay width of the transition $\mu \to e\gamma$ is given by

$$
\Gamma_{\mu \to e\gamma} = \frac{e^2 m_c^2}{\pi} |I(0,0) \text{Re}(g_{\text{L22}}^* g_{\text{R22}})|^2 \left| \frac{g_{\text{L21}}^*}{g_{\text{L22}}^*} \right|^2 m_{\mu}^3
$$

$$
\approx 3.9 \times 10^{-18} \left| \frac{g_{\text{L21}}^*}{g_{\text{L22}}^*} \right|^2 \text{ MeV}.
$$
 (9)

In the above we have used the average value of the muon anomaly reported by the BNL collaboration to estimate $|I(0,0)Re(g_{L22}^*g_{R22})|$. The present experimental bound [7] on the branching ratio for $\mu \to e\gamma$ is 1.2×10^{-11} . It then follows that $|g^*_{L21}/g_{R22}| < 6.2 \times 10^{-5}$. The BNL measurement therefore allows us to determine the scale dependent part of the transition rate unambiguously, leaving only a mixing angle to be constrained from the branching ratio of the rare decay. We find that the relevant flavor violating coupling of the second generation leptoquark must be very strongly suppressed relative to its flavor diagonal coupling.

In conclusion, in this paper we have analyzed the possibility that the BNL muon anomaly is due to a second generation scalar leptoquark that couples to muons of both chiralities. We find that the muon anomaly could arise from relatively light leptoquarks in the few hundred GeV range and couplings that are smaller than the electromagnetic coupling. The bounds are consistent with all other known bounds and in particular the ones arising from helicity suppressed decays of light mesons. We have also shown that the BNL muon anomaly leads to an unambiguous prediction for the EDM of the muon and a bound on the flavor changing leptoquark coupling relevant for the rare decay $\mu \to e\gamma$.

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